Math 10460 - Honors Mathematics II
Homework 8a - Due Wednesday, March 16

(1) (Optional) Show that D_n is a group. *Hint*: a generic element of D_n looks like $g = r^a s^b$ where a and b are integers such that $0 \leq a \leq n-1$ and $s = 0$ or 1. You must show 4 things: multiplying two of these produces a new one (after some simplifications) ($g_1 * g_2 = g_3$), identify an identity element and show it satisfies the properties of an identity ($g * e = g = e * g$), identify an inverse of $g = r^a s^b$ and show it satisfies the properties of an inverse ($g * g^{-1} = e = g^{-1} * g$), and show that the associative property holds ($g_1 * (g_2 * g_3) = (g_1 * g_2) * g_3$) You will need to use the fact that $s^b r^a = r^{(-1)^b a} s^b$ to do this problem.

(2) The usual notation for the set of integers is $\mathbb{Z}$, i.e.,

$$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}.$$

Show that the integers, $\mathbb{Z}$, together with the binary operation given by addition is a group. (You can assume closure (the sum of two integers is an integer) and associativity.)

(3) Is $\mathbb{Z}$ with the binary operation given by multiplication a group? If it is, show it is a group. If it is not, explain why.

(4) In class, we talked about the *orthogonal group* $O(2)$ and the *special orthogonal group* $SO(2)$ as the symmetry group and physical symmetry subgroup, respectively, of the circle.
   (a) There are also the groups $O(3)$ and $SO(3)$. Without looking them up online or in books or anything, what do you think they represent? (Think in terms of symmetry groups and physical symmetry subgroups.)
   (b) Now, look up $O(3)$ and $SO(3)$ somewhere. What do they actually represent? Were you correct in your guess?
   (c) What about the groups $O(n)$ and $SO(n)$? What do they represent? Make a guess before you look it up.